Analysis of the 3D Maxwell-Landau-Lifshitz system of ferromagnetism*

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Abstract

This paper is concerned with regularity of the solutions to the Maxwell-Landau-Lifshitz system describing ferromagnetic medium. We derive estimates for the solution, which can be used to control the Taylor remainder when estimating the error of a numerical scheme. Applications are widely used in the recording industry.
We solve full Maxwell-Landau-Lifshitz (MLL) system in a bounded domain $\Omega \subset \mathbb{R}^3$

$$\partial_t \mathbf{m} = -\mathbf{m} \times (\Delta \mathbf{m} + \mathbf{H}) - \alpha \mathbf{m} \times (\mathbf{m} \times (\Delta \mathbf{m} + \mathbf{H})),$$

$$\begin{aligned} \partial_t \mathbf{E} + \sigma \mathbf{E} - \nabla \times \mathbf{H} &= \mathbf{0}, & \nabla \cdot \mathbf{H} + \beta \nabla \cdot \mathbf{m} &= 0, \\ \partial_t \mathbf{H} + \nabla \times \mathbf{E} &= -\beta \partial_t \mathbf{m}, & \nabla \cdot \mathbf{E} &= 0, \end{aligned}$$

where α, β and σ are physical constants, $\alpha > 0,\ \sigma \geq 0$. We consider Neumann boundary conditions

$$\frac{\partial \mathbf{m}}{\partial \boldsymbol{\nu}}\Big|_{\partial\Omega} = 0, \quad \mathbf{E} \times \boldsymbol{\nu}\big|_{\partial\Omega} = 0, \quad (\mathbf{H} + \beta \mathbf{m}) \cdot \boldsymbol{\nu}\big|_{\partial\Omega} = 0,$$

where $\boldsymbol{\nu}$ is the unit outward normal vector to $\partial\Omega$. The initial conditions read as

$$\mathbf{m}(x,0) = \mathbf{m}_0(x), \quad \mathbf{H}(x,0) = \mathbf{H}_0(x), \quad \mathbf{E}(x,0) = \mathbf{E}_0(x), \quad x \in \Omega \subset \mathbb{R}^3.$$

For the solutions we derive estimates of type

$$\sup_{t \in I} \left\{ \|\partial_t \mathbf{m}\|_2 + \|\partial_t \mathbf{E}\|_2 + \|\partial_t \mathbf{H}\|_2 + \sqrt{\kappa} \|\partial_t \nabla \mathbf{m}\|_2 \right\} + \left(\int_0^{T^*} \|\partial_t \nabla \mathbf{m}(s)\|_2^2 \mathrm{d}s \right)^{1/2} \leq C,$$

$$\Big(\int_0^{T^*} \kappa \{ \|\partial_t^2 \mathbf{m}(s)\|_2 + \|\partial_t \Delta \mathbf{m}(s)\|_2 + \|\partial_t^2 \mathbf{H}(s)\|_2 + \|\partial_t^2 \mathbf{E}(s)\|_2 \}^2 \mathrm{d}s \Big)^{1/2} \leq C,$$

where $\kappa(s) := \min\{s, 1\}$. Such results are essential for rigorous analysis of many numerical schemes. Consider an implicit numerical scheme based on time stepping where time derivatives are approximated by backward Euler approximation. To control the error one takes original equations and subtracts them from the equations defining the numerical scheme. However, one has to deal with the Tailor remainder denoted by $\boldsymbol{R}$ of the form

$$\boldsymbol{R}(\mathbf{m}) := \int_{t_i}^{t_{i+1}} (s - t_i) \partial_t^2 \mathbf{m}(s) \mathrm{d}s.$$

At this stage our estimates can be successfully used to control this remainder.

*This work was supported by the IUAP/V-P5/34 project of Ghent University.
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